Defining Mathematical Giftedness

Linda Parish
Australian Catholic University
<linda.parish@acu.edu.au>

This theoretical paper outlines the process of defining *mathematical giftedness* for a present study on how primary school teaching shapes the mindsets of children who are mathematically gifted. Mathematical giftedness is not a badge of honour or some special value attributed to a child who has achieved something exceptional. Mathematically gifted children possess unusually high natural aptitudes for understanding mathematical concepts, and subsequently differ substantively to their peers in the way they view, understand and learn mathematics.

In preparing to embark on the daunting task of pursuing a doctorate I chose to focus on a long-term passion of mine – children who need additional attention, not because they are struggling to learn, but because they seem to learn so much faster and easier than their age peers: gifted learners. I chose to narrow my focus to mathematical giftedness in order to manage what is a very broad topic, and to build on my recent experience in mathematics education research over the past few years.

It was at this point, however, that I encountered my first dilemma. It seems many people have an aversion to the term *gifted*, so what was I going to call this phenomenon? Some will accept *gifted and talented*; some will simply call them *bright* or *smart* or *exceptional*; others prefer *high achievers* or *highly able* or *highly capable*. I wanted to use a term that was not going to be overtly objectionable at the outset, but at the same time I required a term that accurately describes the children I was wanting to research; a term that could be, at least in part, accessible to the casual reader as well as the academic researcher.

My second dilemma (and possibly related to the first) was how to define the phenomenon. I wasn't necessarily talking about high achievers, or early finishers, or tables-challenge whizzes. It may be easy to spot the child who throws themselves into mathematics classes with enthusiasm and great success, but gifted children are not necessarily high achievers (and sometimes high achievers are not actually gifted). Those children who work quickly and finish set work in half the expected time can be a challenge for their teachers, but gifted children are not necessarily quick workers or 'early finishers'. Children with an exceptional memory for table facts and learned procedures can stand out in some mathematics classes, but these children may simply have good memories and not be particularly good at mathematics at all. Hence my dilemma: what is mathematical giftedness?

Background

Recent research on mathematically gifted children, on identifying them (Bicknell, 2009; Borovik & Gardiner, 2006; Rosario, 2008), on understanding them and how they learn (Leder, 2008), and on providing suitable programs for them (Bicknell, 2009; Diezmann 2005), just to mention a few, provide varying definitions of mathematical giftedness. Borovik and Gardiner (2006) talk about mathematical traits that are both inborn and can "develop and flourish to spectacular effect within a supportive learning

2014. In J. Anderson, M. Cavanagh & A. Prescott (Eds.). Curriculum in focus: Research guided practice *(Proceedings of the 37th annual conference of the Mathematics Education Research Group of Australasia)* pp. 509–516. Sydney: MERGA.

environment” (p. 4). Bicknell (2009) describes mathematically gifted students as those who can be thought of as having “special mathematical abilities, or those who engage in qualitatively different mathematical thinking” (p. 63). Diezman (2005) and Rosario (2008) list characteristics and traits of mathematically gifted students which combine to form a definition of sorts. There is no universally agreed definition of giftedness (Parliament of Victoria, 2012), let alone *mathematical* giftedness. The range of definitions used by researchers to date reflects this.

In order to define mathematical giftedness for my own research I considered it imperative to explore the historical and current notions of ‘giftedness’ so that I could substantiate the terminology I would choose to use, and also to unpack the construct of ‘mathematics’, what it is and how it is recognised, so that it was clear which children I was referring to when talking about the ‘mathematically gifted’.

Giftedness

A Brief History of Giftedness

In 1869 Francis Galton, a British anthropologist best known for his pioneering work on human intelligence, published a book, *Hereditary genius: An inquiry into its laws and consequences*. This was the first social scientific attempt to study ‘genius and greatness in man [sic]’. Galton was interested in identifying genius in order to ‘better society’ through selective breeding. He was a pioneer of the eugenics movement around the turn of the twentieth century. Galton’s definition of genius correlated with ‘man’s [sic] eminence’, and he believed children could inherit the potential to become geniuses (Galton, 1869).

Lewis Terman, an American psychologist who was also a prominent eugenicist, analysed data on ‘mentally superior’ children and in 1906 published his dissertation, *Genius and stupidity: A study of some of the intellectual processes of seven ‘bright’ and seven ‘stupid’ boys* (Terman, 1906). His data became the catalyst of a popular stereotype, that children of superior intelligence are superior in all areas of growth and development, from physical development to emotional wellbeing and health to being more resistant to temptations and juvenile misbehaviour, even though this was never his intent (Whitmore, 1980). Binet and Simon’s standardised scale for ‘intelligence testing’ was developed around this same time (Whitmore, 1980), and while initially intended to discriminate between children who were educable and those who were not, was later revised by Terman, resulting in the Stanford Revision of the Binet-Simon Scale (Terman, Lyman, Ordahl, Ordahl, Galbreath & Talbert, 1917) in order to enable identification of children who were ‘intellectually superior’, a very elitist and audacious epithet. Terman’s studies inadvertently contributed to two of the modern myths about gifted children: 1) that they (and/or their parents) in some way consider themselves ‘superior’, and 2) that they are capable of succeeding on their own without any special assistance (Whitmore, 1980).

The term *gifted*, was first found in psychological literature in the 1920s (Hollingworth, 1926), and was used to delineate children of high intelligence as measured by Terman’s intelligence quotient (IQ) scoring. However, unlike terms such as *idiot*, *imbecile, stupid* and *retarded*, which were acceptable terms to describe children of lower intelligence at this time but have now been replaced with less stigmatizing terms, the term *gifted* has never been successfully ameliorated, even though it has maintained negative connotations of elitism.

Some Popular Modern Models of Giftedness

From the 1970s questions began to arise about the validity of intelligence tests to define giftedness for a number of reasons, not the least of which was that standard IQ tests were very culturally biased, and also because intelligence itself is a difficult-to-define construct (Whitmore, 1980).

In the mid-80s, in defining giftedness, Joseph Renzulli suggested there was an interaction between three traits: general or specific abilities that were above average; commitment to task, or perseverance; and creativity. He called this the *three-ring conception of giftedness*. However, this definition is still limited to measureable achievement: "…it is the interaction among the three clusters that research has shown to be the necessary ingredient for creative-productive accomplishment" (Renzulli, 1998, p. 10). He makes a distinction between the 'gifted' and the 'potentially gifted' suggesting that a person cannot be identified as gifted without an element of all three traits.

In 1985, Robert Sternberg formulated his *triarchic theory of intelligence*, which also takes into consideration three sub-theories: componential intelligence (associated with analytical giftedness), experiential intelligence (associated with creativity or synthetic giftedness), and contextual intelligence (associated with practical giftedness, or 'street smarts') (Sternberg, 1985). Unlike Renzulli, Sternberg believed a gifted person may possess high intelligence in *one or more* of these three intelligence domains. A person's *success* depends on how well these are balanced against each other, but success is not classified as a requirement for giftedness. His view of giftedness presumes characteristics other than simply superior performance on standard intelligence tests. His three loci of intellectual giftedness include dimensions that also take into consideration creativity and the value of a person's culture, something that standard IQ tests do not measure.

At the same time, in 1985, Françoys Gagné introduced a *differentiated model of giftedness and talent* (Gagné, 1991). His conviction was that gifted children do not automatically become productive achievers. Some gifted children (that is, children possessing unusually high natural aptitudes at levels significantly beyond what might be expected for their age) may not necessarily be talented (displaying unusually high levels of achievement or performance) (Gagné, 1991). The term 'gifted and talented' was introduced as a means of differentiating between the 'gift' and the 'talent', and yet popular usage seems to have simply replaced the term 'gifted' with 'gifted and talented'. Talented children are easy to spot. They are the ones achieving great things. Most talented children are also gifted, however not all gifted children are talented (Silverman, 1997). Some gifts remain latent for a whole variety of reasons such as poverty, cultural barriers, learning disabilities, geographical isolation, etc., and also possibly because of teacher expectations of success rather than expectations of effort, which may perpetuate a fear of taking risks due to fear of failure (Dweck, 2006). Surely this is important for educators to be aware of and to explore. *All* children should be encouraged and expected to work to their capabilities, within their zone of proximal development.

In 1991, The Columbus Group – a group of practitioners, parents, and theorists – gathered together to construct a new phenomenological definition of giftedness (Silverman, 1997). Their definition of *asynchronous development* refers to uneven intellectual, physical, and emotional development, which combine to create inner experiences and awareness that are qualitatively different from the norm (Columbus Group, 1991). Asynchronous development places gifted individuals outside normal development patterns from birth to adulthood. Giftedness, then, is viewed as intellectual developmental advancement that can often be observed from very early childhood, not something based purely on school, or academic achievement (Silverman, 1993).

Implications

The history of definitions of giftedness is important in considering the perceptions people have developed of giftedness. Definitions have ranged from associations of superiority and elitism (e.g., Galton and Terman), to sounding like some type of medical condition (e.g., asynchronous development or ‘AS’). This may give some indication as to why the term ‘gifted’ seems to have negative connotations within society, and is something that needs to be addressed if these children are to be seen as students warrantable of special consideration within the education system. If giftedness is defined by talent or accomplishment the argument, ‘Why do we need to cater for gifted children? Aren’t they going to succeed anyway?’ is possibly a valid argument. However, if giftedness is defined as the possession of unusually high natural aptitudes, with a potential that may, or may not be realised due to various life circumstances (including the way these children are being taught), then it is an issue of equity if we are not providing an adequate education for these students. In Australian education today, Gagné’s differentiated model of giftedness and talent provides the most generally accepted definition of giftedness and talent (ACARA, 2011). The Australian Curriculum document recognises that “the school plays a critical role in giving students appropriate opportunity, stimulation and experiences in order to develop their potential and translate their gifts into talents” (ACARA, 2011, para 8).

For my study I have chosen to retain the term ‘gifted’. I recognise that this has negative connotations, and is possibly threatening for some, and I may need to accommodate for individual or school preferences, but I have yet to come up with an alternate term that quite adequately describes the characteristics of the children I am wanting to research. Giftedness indicates an element of unusual inherent ability (Gagné, 1991). If these abilities are unusual, not all children will be gifted: gifted children may be *smart*, but all children are smart to some degree; they may be *highly capable*, but given appropriate learning experiences many children are highly capable; they may be *high achievers*, but as already discussed not necessarily so, and non-gifted children can also be high achievers through effort and hard work. Gifted children may be *exceptional*, but children with learning disabilities are also called exceptional, and giftedness is not a disability or disease. The term *bright* initially seems non-threatening, but if gifted children are bright, by definition what are the implications for non-gifted children? I have come across very few, if any, *dull* children! It may be appropriate to call gifted children *naturally highly capable*, but this is quite a mouthful which may be more suited as a description than as a manageable term. So gifted it is, at least in my formal writings.

Mathematics

What is Mathematics?

I began to realise the extent of the challenge I had posed myself when it dawned on me that not only does the word *gifted* cause many to shudder and put up defences, but so does the word *mathematics*! Mathematics is an often misconstrued subject (Davis, 1984), and my suspicion is that many perceptions of mathematics are built on individual experiences that have oftentimes been somewhat limited. So what is mathematics, beyond the limited school mathematics many have experienced?

Mathematics is an extremely complex construct culminating in logical abstraction of space, form, magnitude and quantity (Davis, 1984). A construction of meaningful mathematical concepts and relationships is not simply a matter of learning and remembering mathematical facts, skills and rules. Mathematics is a creative venture (Sriraman, 2004). It is about problem solving. It is about generalising, extending, creating and deriving ways of approaching new problems (Davis, 1984). Basic mathematics concepts of number, space, measurement, chance, and data are the starting points. Skills of operating with numbers, measuring, describing shapes, drawing graphs, etcetera, are *tools* to enable us to work mathematically; they are a means to an end, not an end in and of themselves.

According to Gardner (1993) logical-mathematical intelligence is about reasoning capabilities, founded on a confrontation with a world of objects that can be grouped, re-grouped and quantified, and moving towards an abstract sense-making of this world of objects, which, while encompassing facility with number, is not limited to number. Gardner’s spatial intelligence also predicates mathematical ability. Spatial intelligence is about the capacity to perceive the visual world accurately, then re-create, transform and/or modify these perceptions mentally (Gardner, 1993).

Sadly for many, experiences of mathematics at school have involved simply the learning of skills (the tools of mathematics), with very little, if any, opportunity to utilise the skills in meaningful and creative ways. To me this is akin to learning to read music notation and understand music theory without ever experiencing the wonder of hearing the “art of sound in time that expresses ideas and emotions in significant forms through the elements of rhythm, melody, harmony, and colour.” (*Dictionary.com*, 2014).

A construction of meaningful mathematical concepts and relationships is not simply a matter of learning and remembering mathematical facts, skills and rules. For my research I assume the characteristics of social interaction, imagery, heuristics, intuition, proof (Sriraman, 2004), generalisation, creation and invention (Krutetskii, 1976) to be as much a part of mathematics as the learning of commonly recognised mathematical skills. Investigation, problem solving, and problem posing, things that require perseverance and sustained effort are necessary for children to learn the ‘art’ and beauty of mathematics.

Mathematically Gifted

Many of the perceived traits of mathematical ability such as rapidity of work, memory of facts and learned procedures are based on a limited understanding of mathematics as mentioned above. These perceptions of mathematical ability have been refuted for years (cf. Krutetskii, 1976) and yet still persist. Vadim Krutetskii’s work in Russia in the 1950s is considered to be landmark work in the study of mathematical ability in school children, and has influenced generations of researchers (Rosario, 2008). According to Krutetskii, children who are mathematically gifted tend to generalise easily, extend, create and invent new methods of solving mathematical problems, and they naturally strive "for the cleanest, simplest, shortest and thus most 'elegant' path to the goal" (Krutetskii, 1976, p. 187).

The mathematically gifted child may be the three year old who can direct you home from wherever you have taken him because of an amazingly instinctual spatial sense; an inbuilt compass, or unusually high spatial intelligence (Gardner, 1993). Or the five-year-old who during play time ‘counts’ 390 unifix cubes by making a 100 mat out of 10 sticks of 10 (Figure 1), and then piling the remaining sticks of 10 on top and patting them down to discover that "… there are 400 cubes [four 'mats' of 100], but there's one stick of 10 missing." Or the seven-year-old who, when shown an array of eight rows of five dots (Figure 2), does not skip count by fives, nor notice four mini-arrays of ten within the array like his peers do, but recognises that eight rows of five is the same quantity as five rows of eight, and hypothesises that this, then, would be "half as many as ten rows of eight". Or the ten-year-old, who has not yet formally learnt ratio, but is able to compare a ratio of 2:3 (A) with a ratio of 3:5 (B) (Figure 3) by recognising the fractional relationships and reasoning that he could compare these "…if you times 3 eighths by 3 you get 9 eighths, which is 1/8 bigger than a whole, and if you times 2 fifths by 3 you get 6 fifths, which is 1/5 bigger than a whole, and 1/5 is bigger than 1/8, so 2/5 is bigger … So A must be more orangey" (Parish, 2010, p. 472).

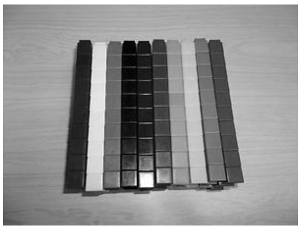

Figure 1.
A 100 mat of unifix cubes

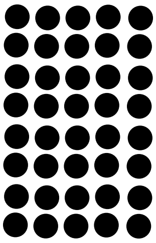

Figure 2.
'8 rows of 5' array

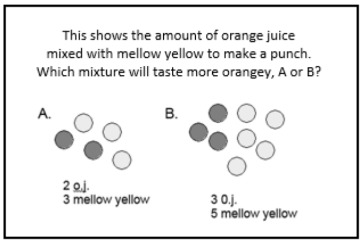

Figure 3.
Ratio problem (Lamon, 1999)

The hallmarks of Krutetskii's mathematically gifted abilities are evident in each of these examples. Generalising, extending, hypothesising, creating new methods … and all for the purpose of making the calculations as simple, and easily reproducible as possible. Can all children be taught to do this? I believe many can. But mathematically gifted children are the ones who are the pioneers of these methods in classrooms. These are the children who discover and/or naturally use these innovative strategies that can be discussed, explored, tested and possibly adopted by others in the class … if the teacher is attuned to the intricacies of what is actually happening mathematically. These are the children who have that innate disposition to think this way, what Krutetskii describes as a "mathematical cast of mind" (Krutetskii, 1976, p. 187).

This unusual aptitude for mathematics is often evident before children start school so an appreciation of these student's needs is important from the very early years of schooling (McAlpine, 2004). Asynchronous development, where a child's intellectual, emotional and physical abilities are not in sync is often evident in children who are intellectually gifted (Silverman, 1997). This means life for these children, especially in the early years, can be quite daunting, and sometimes even frightening (Morelock, 1992; Sword, 2001). This reality is often not recognised due to the persistent popular stereotype perpetrated by Terman (1906) that children of 'superior' intelligence are superior in all areas of growth and development. In order to "make necessary adjustments to meet … individual learning needs [of gifted students]" (ACARA, 2011, para 1) teachers need to be aware of the inner turmoil these children may be facing.

There are some myths of gifted children that need to be dispelled. For example, the notion of gifted children becoming behaviour problems in the classroom because of boredom is a dangerously limited view (Sword, 2001). While this may be true for some, the incidence of introversion is higher in the gifted population compared to the average population (Silverman, 1988). The introverted gifted student is more likely to be overlooked; they are compliant and will therefore do as they are told. Their misery may be internalised, or may come out at home rather than at school (Sword, 2001). Supporting student needs must be valued for the sake of the students' learning. We need to make sure all students who require sufficiently challenging tasks are being catered for, we are not simply providing adequate challenges for students to curb behaviour problems.

Perceived mathematically gifted indicators such as rapidity of work and memory of facts and learned procedures need to be seriously questioned. Many gifted children are not fast workers. Perfectionism is a common characteristic of giftedness (Silverman, 1988), and 'perfection' requires careful attention to detail that can actually be quite slow and laborious. This is an important quality for scientists, surgeons, engineers, computer programmers and so forth, but must be harnessed as a quality for excellence, not a quality for 'accommodation' that results in laborious work on already known mathematics because it is perceived as the 'right thing to do for the teacher' by those compliant, introverted children.

Conclusion

For my research, then, I have chosen to define children who are mathematically gifted as those who *possess unusually high natural (or instinctual) aptitudes for understanding mathematical concepts;* and who therefore *differ substantively to their peers in the way they view, understand and learn mathematics*.

Catering for children who are mathematically gifted is a matter of student equity. These children's learning needs must be recognised, understood and catered for because, as educators, we have a professional obligation to provide for *all* students' needs. "All students regardless of race, age or gender, by virtue of their dignity as human persons, have a right to an education that is suited to their particular needs and adapted to their ability" (Gravissimum Educationis nl, 1965. Declaration on Christian Education, Pope Paul VI) (CEO, 2007, p. 2). "Gifted and talented students are entitled to rigorous, relevant and engaging learning opportunities…aligned with their individual learning needs, strengths, interests and goals" (ACARA, 2011, para 1).

Of course, *defining* mathematical giftedness was only the first step in my PhD journey. The next step was to determine how to *identify* mathematical giftedness. How do you identify such an abstract concept as "a mathematical cast of mind"? But that will have to be my next paper!

References

Australian Curriculum, Assessment and Reporting Authority (ACARA) (2011). *Student Diversity: Gifted and Talented*. Retrieved from http://www.australiancurriculum.edu.au/StudentDiversity/ Gifted-and-talented-students

Bicknell, B. (2009). Who are the mathematically gifted? Student, parent, and teacher perspectives. *Journal of the Korean Society of Mathematical Education, 13*(1), 63-73.

Borovick, A. V., & Gardiner, T. (2006). *Mathematical Abilities and Mathematical Skills.* Paper presented at the World Federation of National Mathematics Competitions Conference, Cambridge, England. Retrieved from http://eprints.ma.man.ac.uk/839/01/covered/MIMS_ep2007_109.pdf

Catholic Education Office (CEO) (2007). *Gifted Education K-12: Position Paper.* Sydney: Catholic Education Office.

Columbus Group (1991, July). Unpublished transcript of the meeting of the Columbus Group. Columbus, Ohio.

Davis, R. B. (1984). *Learning Mathematics: The Cognitive Science Approach to Mathematics Education.* Norwood, NJ: Ablex Publishing Corporation.

Diezmann, C. M. (2005). Challenging mathematically gifted primary students. *Australasian Journal of Gifted Education, 14*(1), 50-57.

Dweck, C. S. (2006). *Mindset: The New Psychology of Success*. New York, NY: Random House.

Gagné, F. (1991). Towards a differentiated model of giftedness and talent. In N. Colangelo & G. A. Davis (Eds.), *Handbook of Gifted Education* (pp. 65-80). Boston, NY: Allyn and Bacon.

Galton, F. (1869). *Hereditary Genius: An Inquiry into its Laws and Consequences*. London: Macmillan and Co.

Gardner, H. (1993). *Frames of Mind: The Theory of Multiple Intelligences* (2nd ed.). New York, NY: Basic Books.

Hollingworth, L. S. (1926). *Gifted Children: Their Nature and Nurture.* New York, NY: Macmillan. Retrieved from http://emilkirkegaard.dk/en/wp-content/uploads/GIFTED-CHILDREN-THEIR-NATURE-AND-NURTURE-Leta-S.-Hollingworth.pdf

Krutetskii, V. A. (1976). *The Psychology of Mathematical Abilities in School Children*. Chicago, IL: University of Chicago Press.

Lamon, S. (1999). *Teaching Fractions and Ratios for Understanding: Essential Content Knowledge and Instructional Strategies for Teachers*. Mahwah, NJ: Lawrence Erlbaum.

Leder, G. (2008). High achievers in mathematics: What can we learn from and about them? In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating Currents and Charting Directions.* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, pp. 195-202). Brisbane, QLD: MERGA.

McAlpine, D. (2004). The identification of gifted and talented children. In D. McAlpine & R, Moltzen (Eds.), *Gifted and Talented: New Zealand Perspectives* (2nd ed., pp. 93-132). Palmerston North: Kanuka Grove Press.

Morelock, M. J. (1992). Giftedness: The view from within. Understanding Our Gifted, 4(3), 1, 11-15.

Music (2014). In *Dictionary.com.* Retrieved from http://dictionary.reference.com/browse/ music?s=t

Parish, L. (2010). Facilitating the development of proportional reasoning through teaching ratio. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the Future of Mathematics Education: Proceedings of the 33rd Annual Conference of the Mathematics Education Research Group of Australasia.* (pp. 469-476). Fremantle: MERGA.

Parliament of Victoria, Education and Training Committee (2012). *Final Report on Inquiry into the Education of Gifted and Talented Students*, Parliament of Victoria, Melbourne. Retrieved from www.parliament.vic.gov.au/images/stories/committees/etc/Past_Inquiries/EGTS_Inquiry/Final_Report/Gifted_and_Talented_Final_Report.pdf

Renzulli J. S. (1998). Three-ring conception of giftedness. In Baum, S. M., Reis, S. M., & Maxfield, L. R. (Eds.). (1998). *Nurturing the Gifts and Talents of Primary Grade Students* (pp. 1-27). Mansfield Center, CT: Creative Learning Press.

Rosario, H. (2008). *Mathematical Minds in Action: Identifying and Nurturing Talent.* Paper presented at the 11th International Congress on Mathematics Education, Monterry, Mexico. Retrieved from http://tsg.icme11.org/tsg/show/7

Silverman, L. K. (1997). The construct of asynchronous development. *Peabody Journal of Education, 72*(3&4), 36-58.

Sriraman, B. (2004). The characteristics of mathematical creativity. *The Mathematics Educator, 14*(1), 19-34.

Sword, L. (2001). Psycho-social needs: Understanding the emotional, intellectual and social uniqueness of growing up gifted. *Gifted & Creative Services Australia Pty Ltd.* Retrieved from http://talentdevelop.com/articles/PsychosocNeeds.html

Terman, L. M. (1906). *Genius and Stupidity*: *A Study of Some of the Intellectual Processes of Seven "Bright" and Seven "Stupid" Boys.* Worcester, MA: Clark University. Retrieved from https://archive.org/details/geniusstupiditys00term

Terman, L. M., Lyman, G., Ordahl, G., Ordahl, L. E., Galbreath, N. & Talbert, W. (1917). *The Stanford Revision and Extension of the Binet-Simon Scale for Measuring Intelligence.* Baltimore, MD: Warwick & York. Retrieved from https://archive.org/details/cu31924030584605

Whitmore, J. R. (1980). *Giftedness, Conflict and Underachievement.* Boston, NY: Allyn and Bacon, Inc.